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Twisted CFT and bilayer Quantum Hall systems in the presence of an impurity

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Abstract

We identify the impurity interactions of the recently proposed CFT description of a bilayer Quantum Hall system at filling $\nu = \frac{m}{pm+2}$ [1]. Such a CFT is obtained by m-reduction on the one layer system, with a resulting pairing symmetry and presence of quasi-holes. For the $m = 2$ case boundary terms are shown to describe an impurity interaction which allows for a localized tunnel of the Kondo problem type.The presence of an anomalous fixed point is evidenced at finite coupling which is unstable with respect to unbalance and flows to a vacuum state with no quasi-holes.

Keyword: Vertex operator, Kac-Moody algebra, Quantum Hall Effect

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1 Introduction

Recently a powerful technique, the m-reduction procedure [2], was successfully employed in [1] in order to construct an effective Conformal Field Theory (CFT) for a Quantum Hall system of two interacting layers at total filling $\nu = \frac{1}{p+1}$. In such a description, the Twisted Model (TM), the neutral current originates from a point-like interaction between the two layers, which can be attributed to the presence of a localized impurity (twist) on the edge. Localized (static) impurities, even though do not change the central charge of the CFT, strongly modify the properties of the ground state and of the spectrum. In fact for any impurities class the Hamiltonian contains a boundary interaction term, which not only determines the ground state energy shift but also gives information on its stability (g-theorem) so that different universality classes exist for a given filling. For a bilayer system, which exhibits interesting phenomena such as interlayer phase coherence [3][4], a few universality classes are known, corresponding to different values of the relevant parameters as the distance between the layers and the symmetric tunneling strength. In a CFT approach they differ in the description of the neutral modes which can be given in terms of symplectic (Haldane-Rezayi model [5]), Dirac (Halperin model [6] (H)) or Ising^2 fermions (TM). While the neutral degrees of freedom contribution to the central charge is $c = -2$ for the first model, for the other two ones its contribution is $c = 1$ and the only difference is in their symmetry. They relate to the Moore-Read (MR) Pfaffian model in which the fundamental particles appear as p-wave BCS like paired states. Pairing symmetry also implies non-Abelian statistics as well as the presence of quasi-holes.

In the TM model the "bulk" degrees of freedom for the bilayer system are described in terms of a free compactified boson X with a $U(1)$ symmetry, for the charged sector, and a Z_2-twisted ϕ field for the neutral ones, at compactification radius $R^2 = 1$ (see [1] for details). On the other hand boundary conformal field theory has been successfully applied to solve quantum impurity problems. By folding a two dimensional system at the defect line, associated with the impurity, the problem is mapped to a system with a boundary. In particular, in [7], starting from the $c = \frac{1}{2}$ Ising CFT, by folding, an (Ising)^2 $c = 1$ CFT is obtained, which can be also described as a Z_2 orbifold of a free boson at compactification radius $R^2 = 1$. In such a boundary CFT, apart for a continuous boundary there are eight mixed boundaries, which are explicitly realized by the Ishibashi states and are fixed at the two ends of the orbifold S^1/Z_2 line.

That appears then a natural framework for studying the stability of the degrees of freedom content of our model (the TM) in terms of boundary states. It is the aim of this letter to express the full set of boundary operators corresponding to the primary fields found in ref. [8]. Indeed we can introduce the chiral partition function $Z_{AB} =< A|e^{-LH}|B >$, where A, B are boundary states (BS), and express it as a superposition of characters of the free bulk CFT. It turns out that the TM characters are in a simple correspondence with the nine possible boundaries of the CFT given in [7]. Furthermore, to convince the reader of the relevance of the TM for the description of a system of highly correlated electrons in the presence of magnetic impurities, we also trace an explicit correspondence between the anisotropic two channel one impurity Kondo problem and the TM (for $p = 0$).

The paper is organized as follows:

In sec. 2 we shortly review the construction of the effective CFT of the TM and its degrees of freedom content in terms of the allowed characters.

In sec. 3 a brief review of the boundary CFT is presented, the Ishibashi states and Cardy consistency conditions recalled and generalized to the orbifold case. The TM characters are then expressed in terms of boundary states.

In sec. 4 the g-stability of the different vacua is analyzed in the framework of the Kondo problem and the identification of the non trivial fixed point (in the Toulouse limit) with the twisted ground state shown.

In sec. 5 a quick summary with comments is presented.

2 The twisted model

In this section we briefly review the m-reduction procedure for the special $m = 2$ case (see ref.[1] for the general case), since we are interested in a system consisting of two parallel layers of 2D electrons gas in a strong perpendicular magnetic field. The filling factor $\nu^{(a)} = \frac{1}{2p+2}$ is the same for the two $a = 1, 2$ layers while the total filling is $\nu = \nu^{(1)} + \nu^{(2)} = \frac{1}{p+1}$. For $p = 0$ ($p = 1$) it describes the 220 (331) Halperin state [6]. The CFT description for such a system can be given in terms of two compactified chiral bosons $Q^{(a)}$ with central charge $c = 2$.

In order to construct the field $Q^{(a)}$ for the TM, let us start from the bosonic "Laughlin" filling $\nu = 1/2(p + 1)$, described by a CFT with $c = 1$ in terms of a scalar chiral field Q compactified on a circle with radius $R^2 = 1/\nu = 2(p + 1)$ (or its dual $R^2 = 2/(p + 1)$). It is explicitly given by:

$$Q(z) = q - i\,p\,lnz + \sum_{n \neq 0} \frac{a_n}{n} z^{-n} \tag{1}$$

with a_n, q and p satisfying the commutation relations $[a_n, a_{n'}] = n\delta_{n,n'}$ and $[q, p] = i$.

The $U(1)$ current $J(z)$ is given by $J(z) = i\partial_z Q(z)$ and the primary fields are expressed in terms of the vertex operators $U^{\alpha}(z) =: e^{i\alpha Q(z)}$: with $\alpha^2 = 1, ..., 2(p + 1)$ and conformal dimension $h = \frac{\alpha^2}{2}$.

From such a CFT (mother theory), using the m-reduction procedure, which consists in considering the subalgebra generated only by the modes in eq.(1) which are a multiple of an integer m, we get a $c = m$ orbifold CFT (daughter theory, i.e. the TM) which describes the LLL dynamics. Then the fields in the mother CFT can be organized into components which have well defined transformation properties under the discrete Z_m (twist) group, which is a symmetry of the TM. By using the mapping $z \rightarrow z^{1/m}$ and by making the identifications $a_{nm+l} \longrightarrow \sqrt{m} a_{n+l/m}$, $q \longrightarrow \frac{1}{\sqrt{m}} q$ (see ref. [9]) the $c = m$ CFT (daughter theory) is obtained.

Its primary fields content, for the special $m = 2$ case, can be expressed in terms of:

1) the Z_2-invariant scalar field $X(z)$, given by

$$X(z) = \frac{1}{2}\left(Q^{(1)}(z) + Q^{(2)}(-z)\right) \tag{2}$$

corresponding to a boson compactified on a circle with radius R_X now equal to $R_X^2 = R^2/2 = p + 1$, which describes the electrically charged sector of the new filling;

2) the twisted field

$$\phi(z) = \frac{1}{2}\left(Q^{(1)}(z) - Q^{(2)}(-z)\right) \tag{3}$$

which satisfies the twisted boundary conditions $\phi(e^{i\pi} z) = -\phi(z)$ and describes the neutral sector [1]. Notice that its compactification radius is $R_\phi^2 = 1$ independent on the flux p.

The chiral fields $Q^{(a)}$, defined on a single layer $a = 1, 2$, due to the boundary conditions imposed upon them by the orbifold construction, can be thought as components of a unique "boson" defined on a double covering of the disc (layer) ($z_i^{(1)} = -z_i^{(2)} = z_i$). As a consequence of such a construction the two layers system becomes equivalent to one-layer QHF (in contrast with the Halperin model in which they appear independent) and the X and ϕ fields defined in eqs. (2) and (3) diagonalize the interlayer interaction. In particular, the X field carries the total charge with velocity v_X while ϕ carries the charge difference of the two edges with velocity v_ϕ i.e. no charge, being the number of electrons the same for each layer (balanced system) [10].

Correspondingly the Virasoro generator splits into the two terms [9]:

$$T_X(z) = -\frac{1}{2}\left(\partial_z X(z)\right)^2 \tag{4}$$

and

$$T_\phi(z) = -\frac{1}{4}\left(\partial_z \phi(z)\right)^2 + \frac{1}{16z^2} \tag{5}$$

each contributing with $c = 1$ to the central charge. The primary fields are composite operators and, on the torus, they are described in terms of the conformal blocks of the MR and the Ising model [8]. The MR characters $\chi^{MR}_{(\lambda,s)}$ with $\lambda = 0, ...2$ and $s = 0, ..., p$, are explicitly given by:

$$\begin{aligned}
\chi^{MR}_{(0,s)}(w|\tau) &= \chi_0(\tau) K_{2s}(w|\tau) + \chi_{\frac{1}{2}}(\tau) K_{2(p+s)+2}(w|\tau) && (6)\\
\chi^{MR}_{(1,s)}(w|\tau) &= \chi_{\frac{1}{16}}(\tau)\left(K_{2s+1}(w|\tau) + K_{2(p+s)+3}(w|\tau)\right) && (7)\\
\chi^{MR}_{(2,s)}(w|\tau) &= \chi_{\frac{1}{2}}(\tau) K_{2s}(w|\tau) + \chi_0(\tau) K_{2(p+s)+2}(w|\tau). && (8)
\end{aligned}$$

They represent the field content of the Z_2 invariant $c = 3/2$ CFT [11] with a charged component ($K_\alpha(w|\tau) = \frac{1}{\eta(\tau)}\Theta\left[\begin{array}{c}\frac{\alpha}{4(p+1)}\\ 0\end{array}\right](2\,(p+1)\,w|4\,(p+1)\,\tau)$) and a neutral component (χ_β, the conformal blocks of the Ising Model).

The characters of the twisted sector are given by:

$$\chi^{+}_{(0,s)}(w|\tau) = \bar{\chi}_{\frac{1}{16}}\left(\chi^{MR}_{(0,s)}(w|\tau) + \chi^{MR}_{(2,s)}(w|\tau)\right) \tag{9}$$

$$\chi^{+}_{(1,s)}(w|\tau) = \left(\bar{\chi}_0 + \bar{\chi}_{\frac{1}{2}}\right)\chi^{MR}_{(1,s)}(w|\tau) \tag{10}$$

which do not depend on the parity of p;

$$\chi^{-}_{(0,s)}(w|\tau) = \bar{\chi}_{\frac{1}{16}}\left(\chi^{MR}_{(0,s)}(w|\tau) - \chi^{MR}_{(2,s)}(w|\tau)\right) \tag{11}$$

$$\chi^{-}_{(1,s)}(w|\tau) = \left(\bar{\chi}_0 - \bar{\chi}_{\frac{1}{2}}\right)\chi^{MR}_{(1,s)}(w|\tau) \tag{12}$$

for p even, and

$$\chi^{-}_{(0,s)}(w|\tau) = \bar{\chi}_{\frac{1}{16}}\left(\chi_0 - \chi_{\frac{1}{2}}\right)\left(K_{2s}\,(w|\tau) + K_{2(p+s)+2}\,(w|\tau)\right) \tag{13}$$

$$\chi^{-}_{(1,s)}(w|\tau) = \chi_{\frac{1}{16}}\left(\bar{\chi}_0 - \bar{\chi}_{\frac{1}{2}}\right)\left(K_{2s+1}\,(w|\tau) - K_{2(p+s)+3}\,(w|\tau)\right) \tag{14}$$

for p odd.

Notice that the last two characters are not present in the TM partition function and that only the symmetric combinations $\chi^{+}_{(i,s)}$ can be factorized in terms of the $c = \frac{3}{2}$ and $c = \frac{1}{2}$ theory. That is a consequence of the parity selection rule (m-ality), which gives a gluing condition for the charged and neutral excitations.

Furthermore the characters of the untwisted sector are given by:

$$\tilde{\chi}^{+}_{(0,s)}(w|\tau) = \bar{\chi}_0\chi^{MR}_{(0,s)}(w|\tau) + \bar{\chi}_{\frac{1}{2}}\chi^{MR}_{(2,s)}(w|\tau) \tag{15}$$

$$\tilde{\chi}^{+}_{(1,s)}(w|\tau) = \bar{\chi}_0\chi^{MR}_{(2,s)}(w|\tau) + \bar{\chi}_{\frac{1}{2}}\chi^{MR}_{(0,s)}(w|\tau) \tag{16}$$

$$\tilde{\chi}^{-}_{(0,s)}(w|\tau) = \bar{\chi}_0\chi^{MR}_{(0,s)}(w|\tau) - \bar{\chi}_{\frac{1}{2}}\chi^{MR}_{(2,s)}(w|\tau) \tag{17}$$

$$\tilde{\chi}^{-}_{(1,s)}(w|\tau) = \bar{\chi}_0\chi^{MR}_{(2,s)}(w|\tau) - \bar{\chi}_{\frac{1}{2}}\chi^{MR}_{(0,s)}(w|\tau) \tag{18}$$

$$\tilde{\chi}_{(s)}(w|\tau) = \bar{\chi}_{\frac{1}{16}}\chi^{MR}_{(1,s)}(w|\tau) \tag{19}$$

We point out that the periodic sector of the TM describes a model for the bilayer which was introduced by Ho in [12]. In [1] it was observed that this sector has degeneracy $3(p+1)$ instead of the $4(p+1)$ one of the Halperin model. The difference between the Halperin and the Ho description relies in the different strength of the Coulomb interaction between the electrons in the same layer with respect to that between electrons in different ones. From the CFT point of view these effects modify the neutral sector from a Dirac to an Ising^2 fermionic theory and it is a consequence of the Abelian orbifold considered there. Moreover the CFT for the Ho model, which was introduced to describe a continuous transition to a Pfaffian MR state due to the degeneracy matching, is inconsistent without the twisted sector of the TM (see [1]). The reduction of the degrees of freedom is basically due to the Z_2 symmetry of the TM and is closely related to its non-Abelian statistics. That seems to be a peculiarity of the TM with respect to the unorbifolded H model. Furthermore the H model has an $U(1)$ symmetry which is explicitly broken in the TM and an observable phase can be induced by the continuous Dirichlet BS introduced in the next section in eq.(33). As a consequence a phase coherence between the particles on the two layers takes place and a Josephson like effect is possible [13].

3 Boundary CFT and the twisted model

Homogenous systems are in many cases idealizations of real systems, which in general present disomogeneities or defects. Boundary Conformal Field Theory (BCFT) has been developed in the last few years to solve such problems. In particular an interesting relationship has been traced between the BCFT and the Kondo effect, where the interaction energy of the conduction electrons with the quantum impurity internal degrees of freedom results, at large distances, into a boundary condition for the fields describing the electrons in the bulk. Furthermore such a non trivial boundary condition in most cases turns out to be conformally invariant. More recently BCFT has been applied to the two dimensional Ising model with a defect line [7]. By folding the system at the defect line, a mapping can be made to a boundary problem and the BS can be identified in the $c = 1$ Z_2 orbifold CFT of a free boson. The construction of the BS goes basically through two consistency requirements: the Ishibashi and the Cardy conditions [14]. In order to classify boundary conditions let us consider a cylinder with a periodic direction of length $1/T$ and of length L in the axis direction. If we indicate with A, B the boundary conditions on the cylinder edges we can evaluate the partition function Z_{AB} for such a configuration in two ways:

- $Z_{AB} = tr\ e^{-\frac{H_{AB}}{T}}$ - that is the cylinder surface may be viewed as swept by an open string (with boundary conditions A, B at its end points) which propagates in a closed loop of length $1/T$, H_{AB} being its Hamiltonian depending on the boundary conditions A, B;
- $Z_{AB} =< A|e^{-LH}|B >$ - that is the closed string boundary state $|B >$ propagates along L with a periodic Hamiltonian H turning into a boundary state $|A >$.

The corresponding expressions are given in terms of characters of the bulk theory: $Z_{AB} = \sum_{ij} n^i_{AB} S^j_i \chi_j(q)$, where S gives the modular transformations of the characters χ and n^i_{AB} is the number of times the irreducible representation of highest weight i appears in the spectrum of the crossed-channel Hamiltonian between the boundary A and B.

The BS must satisfy the condition: $\left(L_n - \bar{L}_{-n}\right)|B >= 0$, required by conformal invariance or a stronger condition $(a_n \pm \bar{a}_{-n})\,|B >= 0$ where a_n, $\bar{a}_{-n}$ are free oscillators.

The BS satisfying the above equation (Dirichlet for the $-$ sign, Neumann for the $+$ sign) are defined up to a normalization constant which is fixed by the Cardy condition rising from equating Z_{AB} in two descriptions given before. The duality between the electric charge and the magnetic flux also exchange Dirichlet and Neumann boundary conditions. The folding procedure is used in the literature to map a problem with a defect line (as a bulk property) into a boundary one, where the defect line appears as a boundary state of a theory which is not anymore chiral and its fields are defined in a reduced region which is one half of the original one. Our approach in [1] is a chiral description of that, where the chiral ϕ field defined in $(-L/2,\ L/2)$ describes both the left moving component ϕ_L and the right moving one ϕ_R defined in $(-L/2,\ 0)$, $(0,\ L/2)$ respectively, in the folded description. Furthermore to make a connection with the TM we consider more general gluing conditions:

$$\phi_L(x=0) = \mp\phi_R(x=0) - \varphi_0$$

the $-(+)$ sign staying for the twisted (untwisted) sector. We are then allowed to use the boundary states given in [7] for the $c = 1$ orbifold at the Ising^2 radius. The X field, which is even under the folding procedure, does not suffer any change in boundary conditions and remains always in the Dirichlet state.

The orbifold nature of this CFT allows us to make contact with the BCFT as described in [7] for the $m = 2$ case. In this case the defect lines in the Ising model can be seen as BS in the folded model (i.e. the $c = 1$ orbifold or equivalently Ising^2). They are given in terms of the Ising ones as:

$$|\uparrow> \;=\; \frac{1}{\sqrt{2}}(||I>> + ||\epsilon>>) + \frac{1}{2^{1/4}}||\sigma>> \tag{20}$$

$$|\downarrow> \;=\; \frac{1}{\sqrt{2}}(||I>> + ||\epsilon>>) - \frac{1}{2^{1/4}}||\sigma>> \tag{21}$$

$$|f> \;=\; ||I>> - ||\epsilon>> \tag{22}$$

where $||I>>$, $||\epsilon>>$ and $||\sigma>>$ are the standard Ishibashi states of the Ising model [15]. The partition function of an Ising model with defects has been considered in [16]. There are three independent possibilities:

$$Z_{0,\frac{1}{2}} = (\bar{\chi}_0\chi_{\frac{1}{2}} + c.c.) + \bar{\chi}_{\frac{1}{16}}\chi_{\frac{1}{16}} \tag{23}$$

$$Z_{\frac{1}{16},0} = \bar{\chi}_{\frac{1}{16}}(\chi_0 + \chi_{\frac{1}{2}}) + c.c \tag{24}$$

$$Z_{\frac{1}{16},\frac{1}{16}} = Z_{0,\frac{1}{2}} + Z_{\frac{1}{16},0} \tag{25}$$

In the orbifold model there are nine BS given by the product of the two Ising BS and two continuous ones which cannot be related to the Ising model. These boundaries fall into 3 classes with different boundary entropy $g_I = 1,\ 1/2,\ 1/\sqrt{2}$ respectively and their stability under the boundary perturbation is obtained according to the "g theorem", with g decreasing along a renormalization group trajectory connecting two conformally invariant boundary conditions. Nevertheless, while in [7] the two Ising are equivalent in our case the coupling to the charged sector of the even Ising breaks this Z_2 symmetry.

The Ishibashi states for the TM are easily obtained as combinations of the BS of the charged and neutral sector.

The most convenient representation of such BS is the one in which they appear as a product of Ising and MR BS:

$$|\chi^{MR}_{(0,s)}>=|2s>\otimes|\uparrow>+|2(s+p)+2>\otimes|\downarrow> \tag{26}$$

$$|\chi^{MR}_{(1,s)}>=\frac{1}{2^{1/4}}\left(|2s+1>+|2(s+p)+3>\right)\otimes|f> \tag{27}$$

$$|\chi^{MR}_{(2,s)}>=|2s>\otimes|\downarrow>+|2(s+p)+2>\otimes|\uparrow> \tag{28}$$

Such a factorization naturally arises already for the TM characters [8]. In the above equation the MR BS are given in terms of the BS $|\alpha>$ for the charged boson (see ref.[15] for details) and the Ising ones.

The vacuum state for the TM model corresponds to the $\tilde{\chi}_{(0,0)}$ character which is the product of the vacuum state for the MR model and that of the Ising one. As we can see in eqs.(15,17) the lowest energy state appears in two characters. As it has been already shown that is a characteristics of the orbifold construction and a linear combination of them must be taken in order to define a unique vacuum state (see ref.[17] for a generalization of the Cardy condition). So the correct BS in the untwisted sector are:

$$|\tilde{\chi}_{((0,s),0)}>=\frac{1}{\sqrt{2}}\left(|\tilde{\chi}^{+}_{(0,s)}>+|\tilde{\chi}^{-}_{(0,s)}>\right)=\sqrt{2}(|2s>\otimes|\uparrow\bar{\uparrow}>+|2(s+p)+2>\otimes|\downarrow\bar{\uparrow}>) \tag{29}$$

$$|\tilde{\chi}_{((0,s),1)}>=\frac{1}{\sqrt{2}}\left(|\tilde{\chi}^{+}_{(0,s)}>-|\tilde{\chi}^{-}_{(0,s)}>\right)=\sqrt{2}(|2s>\otimes|\downarrow\bar{\downarrow}>+|2(s+p)+2>\otimes|\uparrow\bar{\downarrow}>) \tag{30}$$

$$|\tilde{\chi}_{((1,s),0)}>=\frac{1}{\sqrt{2}}\left(|\tilde{\chi}^{+}_{(1,s)}>+|\tilde{\chi}^{-}_{(1,s)}>\right)=\sqrt{2}(|2s>\otimes|\downarrow\bar{\uparrow}>+|2(s+p)+2>\otimes|\uparrow\bar{\uparrow}>) \tag{31}$$

$$|\tilde{\chi}_{((1,s),1)}>=\frac{1}{\sqrt{2}}\left(|\tilde{\chi}^{+}_{(1,s)}>-|\tilde{\chi}^{-}_{(1,s)}>\right)=\sqrt{2}(|2s>\otimes|\uparrow\bar{\downarrow}>+|2(s+p)+2>\otimes|\downarrow\bar{\downarrow}>) \tag{32}$$

$$|\tilde{\chi}_{(s)}(\varphi_0)>=\frac{1}{2^{1/4}}\left(|2s+1>+|2(s+p)+3>\right)\otimes|D_0(\varphi_0)> \tag{33}$$

where we also added the states $|\tilde{\chi}_{(s)}(\varphi_0)>$ in which $|D_0(\varphi_0)>$ is the continuous orbifold Dirichlet boundary state defined in ref.[7]. For the special $\varphi_0=\pi/2$ value one obtains:

$$|\tilde{\chi}_{(s)}>=\frac{1}{2^{1/4}}\left(|2s+1>+|2(s+p)+3>\right)\otimes|ff> \tag{34}$$

We briefly discuss the relevance of such a continuous state in sec.(5).

For the twisted sector we have:

$$|\chi^{+}_{(0,s)}>=(|2s>+|2(s+p)+2>)\otimes(|\uparrow\bar{f}>+|\downarrow\bar{f}>) \tag{35}$$

$$|\chi^{+}_{(1,s)}>=\frac{1}{2^{1/4}}(|2s+1>+|2(s+p)+3>)\otimes(|f\bar{\uparrow}>+|f\bar{\downarrow}>) \tag{36}$$

On the other hand the modular transformations for the characters $\chi^{-}_{(i,s)}$ depend on the parity of p, and as a consequence the BS also depends on it:

for p even

$$\begin{aligned}|\chi^{-}_{(0,s)}\ \ > &= (|2s>-|2(s+p)+2>)\otimes(|\uparrow\bar{f}>-|\downarrow\bar{f}>) && (37)\\ |\chi^{-}_{(1,s)}\ \ > &= \frac{1}{2^{1/4}}(|2s+1>+|2(s+p)+3>)\otimes(|f\bar{\uparrow}>-|f\bar{\downarrow}>) && (38)\end{aligned}$$

for p odd

$$\begin{aligned}|\chi^{-}_{(0,s)}\ \ > &= (|2s>+|2(s+p)+2>)\otimes(|\uparrow\bar{f}>-|\downarrow\bar{f}>) && (39)\\ |\chi^{-}_{(1,s)}\ \ > &= (|2s+1>-|2(s+p)+3>)\otimes(|f\bar{\uparrow}>-|f\bar{\downarrow}>) && (40)\end{aligned}$$

In order to compare the relative stability of the previous boundary states, let us define the g entropy function for the TM. In general, under boundary RG flow, the central charge c stays fixed. The flow takes place in the space of boundary conditions of the bulk theory and a quantity analogous to c can be defined, that is the boundary entropy g [18]. It measures the "number of boundary degrees of freedom" and decreases along the RG flow. Such a quantity can be defined as the term in the annulus partition function which is independent of the width L of the strip in the thermodynamic $L\to\infty$ limit or, equivalently, as the disk partition function $g_B =< 0||B>$ of the boundary state $|B>$ associated to the perturbed theory. In our case it can be explicitly evaluated as $g_a =< \tilde{\chi}_{((0,0),0)}||\chi_a>$ for any boundary state $|\chi_a>$. Furthermore these BS satisfy a generalized Cardy condition [17] and the g function is expressed as $g = g_{MR}g_{\bar{I}}$ where $g_{MR}=\sqrt{2(p+1)}\sin\frac{\pi}{4}(\lambda+1)$.

By using the vacuum state given in eq.(29) we found the following values of g for the different classes of boundary conditions:

TM	g
$\lvert\tilde{\chi}_{((i,s),f)}>$	$\frac{\sqrt{p+1}}{2}$
$\lvert\tilde{\chi}_{(s)}>$	$\sqrt{2(p+1)}$
$\lvert\chi^{\pm}_{(i,s)}>$	$\sqrt{\frac{p+1}{2}}$

4 Twisted BS and stability

In order to understand more on the stability of the BS just given before, it is crucial to view our approach within the framework of the Kondo and related problems. The low T behavior of a system in the presence of an impurity is described by an effective CFT in which the impurity disappears (it is screened) but certain interactions between the quasi-particles are generated in the screening process. We first discuss the class of impurities in the even interaction channel which produces a two-channel Kondo like interaction. In this case only the symmetric boundary terms give contribution to the dynamics. In general we can introduce two parameters V_1 and V_2 for the boundary potential for the up and down layer respectively which can be also written in the even (odd) basis as: $V = (V_1 + V_2)/2$ ($\bar{V} = (V_1 - V_2)/2$). The relevant quantum number λ_I of the impurity gets hybridized with that of the fluid (see the definition of λ in sec.(3)), as it appears in the boundary state, λ_I being equivalent to the $su(2)$ spin of the Kondo problem [3]. The $\lambda_I = 0$ value corresponds to the free case (no interactions between the Hall fluid electrons and the impurity), while $\lambda_I = 1$ and $\lambda_I = 2$ correspond to the overscreened and the exactly-screened Kondo interaction respectively. From the QHE point of view λ_I describes a paired (λ_I even) or unpaired (λ_I odd) impurity. An impurity with $\lambda_I = 1$ induces a breaking of the pairing symmetry and a flow which changes the boundary conditions. Indeed our description adapts very closely to a system of two interacting Luttinger liquids coupled resonantly through an impurity placed in between. Such a system is described by the tunneling term:

$$H_V = V\left(\cos\alpha X(0)\cos\phi(0)d_x^{\lambda_I} + \sin\alpha X(0)\cos\phi(0)d_y^{\lambda_I}\right) - \tilde{V}d_z^{\lambda_I}\partial X(0) \tag{41}$$

where $\alpha = \sqrt{2/(p+1)}$ and V, $\tilde{V} = \left(\tilde{V}_1 + \tilde{V}_2\right)/2$ are coupling constants and $d_x^{\lambda_I}, d_y^{\lambda_I}, d_z^{\lambda_I}$ ($\lambda_I = 1$ here) describe the impurity "spin" placed at the origin. The above Hamiltonian is similar to that of the closely related problem of a resonant tunneling junction between quantum wires or hopping of electrons in Quantum Hall bars coupled to a dot or anti-dot (see ref.[19] for details). In the framework of the Kondo problem such an interaction term is equivalent to the anisotropic two-channel one impurity Hamiltonian [19] and for $V = \tilde{V}$ it reduces to the isotropic case. The $V = 0, \pm\infty$ conformal fixed points are unstable and flow to an intermediate stable point V^*. For the isotropic Kondo problem, i.e. $p = 0$, and m channels it is always possible to "complete the square" at the special value $V^* = \frac{2}{2+m}$, where the complete Hamiltonian reduces to its free form after a shift of the current operators by $\mathbf{d}^{\lambda_I}$ which preserves the $su(2)_m$ Kac-Moody algebra. That corresponds to the "absorption" of the impurity and in such a case the system renormalizes to the intermediate fixed point V^*("fusion rules hypothesis"[18]).

In the more general $V \neq \tilde{V}$ case instead, by performing the rotation $U = e^{i\tilde{V}d_z^{\lambda_I}X(0)}\sigma$, ($\sigma$ being the twist operator of the Ising model) H_V reduces to $H_V = V_{eff}\cos\phi(0)d_x^{\lambda_I} -$

[3]Notice that the quantum number s defined in sec.(3), describing flux addition, plays no role in the stability of the ground state and then can be taken equal to zero. In such a case a diagonal form of the partition function is recovered.

$\tilde{V} d_z^{\lambda_I} \partial X(0)$. After such a transformation the boundary conditions for both the charged and neutral bosons are changed according to the "fusion principle" (see ref. [7]). The X boson acquires a phase shift at the impurity location while the neutral one gets both twisted and shifted. These new boundary conditions are consistent with the parity rule for the bulk CFT discussed in [8]. The boundary equation of motion can be obtained by varying the action with respect to all the fields, including $d_x^{\lambda_I}$,$d_y^{\lambda_I}$, which are dynamical variables. The vacuum states corresponding to these fixed points belong to the twisted sector as it can be easily understood by noticing that the U operator corresponds to the $\lambda = 1$ representation of the MR model. The total dimension of U is: $h = \frac{1}{8(p+1)} + \frac{1}{16} = \frac{3+p}{16(p+1)}$. At the Toulouse point the last term cancels out with a term generated by the rotation U in the free Hamiltonian obtaining $H_V = V_{eff} \cos\phi(0) d_x^{\lambda_I}$, with the resulting decoupling of the charged field X from the neutral field ϕ. Notice that the above transformation changes the reference state from $< \tilde{\chi}_{((0,0),0)}|$ to $< \chi^+_{(1,0)}|$ (see eqs.(29,36)) and at the same time the bare coupling constant (V) appearing in eq.(41) is mapped to the new effective one (V_{eff}). In fact from the RG equation for the boundary entropy a stable fix point is reached at a finite value of V which is mapped into an infinite V_{eff} fixed point after the U rotation. As for the isotropic case, the new interaction becomes that of a $\lambda_I = 1$ exactly-screened problem.

In particular to the $V_{eff} = 0$ value corresponds the unstable boundary state $|\tilde{\chi}_{(s)} >$ with $g = \sqrt{2(p+1)}$ while to the $V_{eff} = \pm\infty$ limit the stable $|\chi^+_{(1,s)} >$ ones with $g = \sqrt{\frac{p+1}{2}}$. In such ground states only one component of the impurity enters the Hamiltonian while the other one describes non dynamical fermionic degrees of freedom. That describes a non-Fermi liquid fixed point for the neutral sector which generalizes the $p = 0$ case.

As it is well known for the Kondo problem, under a flavour symmetry breaking perturbation the fixed point is unstable [20]. An asymmetric local tunneling term in the Hamiltonian induces a flow to one of the two stable points in which the impurity hybridizes with one channel only. The same structure exists in the present system. A schematic RG flow in terms of the two coupling constants V_1, V_2 of the layers is given in fig:(1).

The new perturbation gives rise to a flow from the unstable intermediate fixed point V^* to the states given by $|\tilde{\chi}_{(0,s)} >$, $|\tilde{\chi}_{(1,s)>}$, which are the most stable ones with a value of g given by $g = \frac{\sqrt{p+1}}{2}$. That is close to the case of a one-channel Kondo problem for the layer interacting with the impurity while the other one remains completely free. We observe that no quasi-holes are present in the stable state and by taking a linear combination it is easy to see that there is a complete factorization of the charged and neutral degrees of freedom, that is Abelian statistics is recovered!

It is worthwhile to comment briefly on the cocycles here, which have to be handled with great care in problems with several fermion species. In addition to the exponential of a free boson, each fermion requires a cocycle. We also notice that the Kondo analysis should be restricted to the fermionic case (i.e. for p odd) for which the Pauli principle is at work. However that does not modify our analysis.

5 Summary and comments

As it has been seen in this letter, the possibility of viewing the TM degrees of freedom in terms of BS has allowed us to establish a close relation of the twisted theory, previously used for describing a Quantum Hall system of two interacting layers, with the two-channel one impurity Kondo problem. In such a context the different possibilities for the bulk electrons of exactly-screening or overscreening the impurity spin have been identified. On the other hand such an identification has allowed us to understand the degree of stability of the different TM boundary states. In particular it has been shown that the two layers resonant case corresponds to the situation in which the impurity spin couples with the same strength to the electrons of the two different channels, i.e. the layers in the Quantum Hall bilayer ($V_1 = V_2$). Then the two layers are completely balanced and the ground state shows non-Abelian statistics due to the existence of quasi-holes states. From the analysis of the flow of the boundary entropy g such a vacuum is not stable under antisymmetric perturbations (it is a saddle point). For strong coupling of the impurity spin with the bulk electrons belonging to only one of the two layers ($|V_1|$ or $|V_2| \to \infty$) the system degenerates towards the most stable ground state with no quasi-holes to condense, the degrees of freedom left being only electrons and anyons states with Abelian statistics. Therefore we conjecture that non-Abelian statistics can be realized only in an extremely clean sample while in the presence of impurities the statistics reduces to the Abelian one. Furthermore the twist at the origin couples the two layers in a topological way, with a consequent phase shift between electrons in the up and down layer which is constant. Such a phase difference fixed to π for the twisted sector ground state, can be deformed to a continuous value φ_0 in the boundary state $|\tilde{\chi}_{(s)}(\varphi_0) >$ allowing for a Josephson like effect, which is at the moment under study. Another interesting problem to study is the case in which there are both boundary and bulk perturbations. In this case the flow can act also on the central charge of the CFT and the reduction to a pure MR model could be realized. Due to the analogy with the string and D-brane dynamics, the present analysis also applies to D-branes systems as it was done in [21]. Finally we notice that for an impurity with quantum number $s_I \neq 0$ the scattered particle might change statistics and at the same time the partition function would be in a non-diagonal form.

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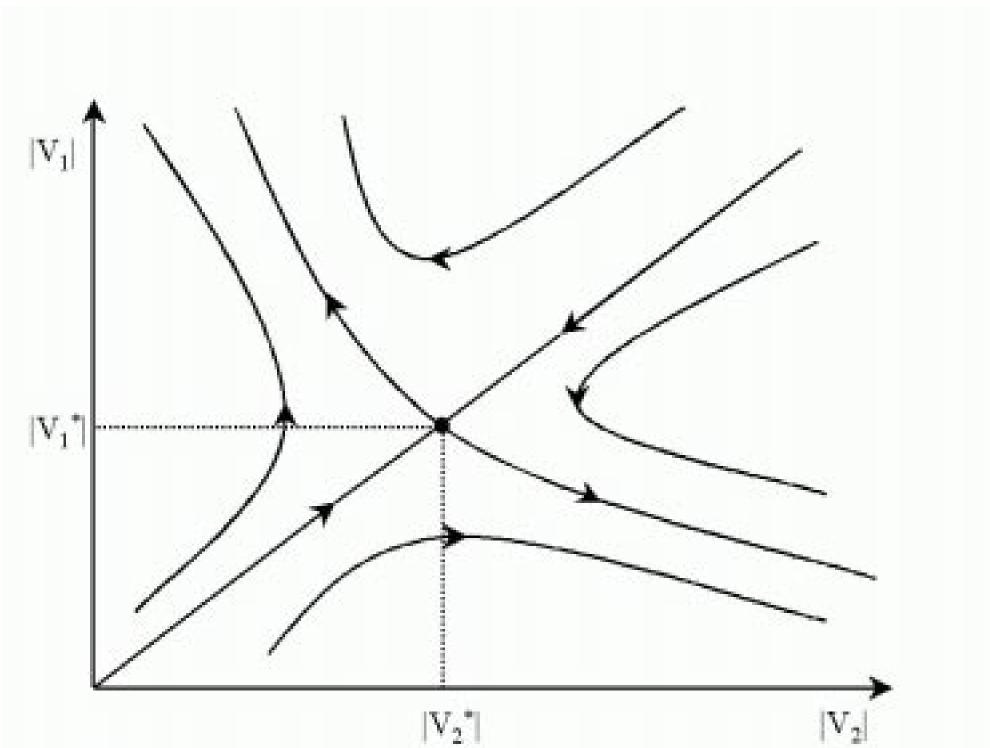

Figure 1: A schematic RG flow diagram for the TM in the V_1, V_2 parameters space.